ECEN 360
Final Exam Appendix

Maxwell's Equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \cdot \vec{D} = \rho_v$$

$$\vec{D} = \varepsilon \vec{E} = \varepsilon_r \varepsilon_o \vec{E}$$

$$\vec{B} = \mu \vec{H} = \mu_r \mu_o \vec{H}$$

$$\vec{J} = \sigma \vec{E}$$

Gauss's Law: $\oint_S \vec{D} \cdot d\vec{s} = \int_v \rho_v \, dv = Q$

Voltage: $V = -\int_l \vec{E} \cdot d\vec{l}$

Ampere's Law: $\oint_C \vec{H} \cdot d\vec{l} = \int_s \vec{J} \cdot d\vec{s} = I$

Magnetic flux: $\Phi = \int_s \vec{B} \cdot d\vec{s}$

Boundary Conditions

Tangential E:	$E_{1t} = E_{2t}$	$\hat{n} \times \left(\vec{E}_1 - \vec{E}_2 \right) = 0$
Normal D:	$D_{1n} - D_{2n} = \rho_s$	$\hat{n} \cdot \left(\vec{D}_1 - \vec{D}_2 \right) = \rho_s$
Tangential H:	$H_{1t} - H_{2t} = J_s$	$\hat{n} \times \left(\vec{H}_1 - \vec{H}_2 \right) = \vec{J}_s$
Normal B:	$B_{1n} - B_{2n} = 0$	$\hat{n} \cdot \left(\vec{B}_1 - \vec{B}_2 \right) = 0$

Faraday's Law: $V = -N \frac{\partial}{\partial t} \int_S \vec{B} \cdot d\vec{s}$

Calculus Theorems

Divergence Theorem: $\int_V \nabla \cdot \vec{F} \, dv = \oint_S \vec{F} \cdot d\vec{s}$

Stokes's Theorem: $\int_S \left(\nabla \times \vec{F} \right) \cdot d\vec{s} = \oint_C \vec{F} \cdot d\vec{l}$

Material Parameters

Permittivity of free-space: ε_o=8.85 X 10^{-12} F/m

Permeability of free-space: μ_o=4π x 10^{-7} H/m

Plane Wave

General propagation constant:
$$\gamma^2 = -\omega^2 \mu \varepsilon_c = -\omega^2 \mu\left(\varepsilon_o \varepsilon_r - j\frac{\sigma}{\omega}\right) = -\omega^2 \mu \varepsilon_c$$
$$\gamma = \alpha + j\beta$$

General plane wave equations:
$$\vec{E} = \vec{E}_o\, e^{-\gamma z} \quad \text{or} \quad \vec{E} = \vec{E}_o\, e^{-jkz}$$
$$\vec{E} = -\eta\, \hat{k} \times \vec{H}$$
$$\vec{H} = \frac{1}{\eta}\, \hat{k} \times \vec{E}$$

Lossless propagation constant: $k = \omega\sqrt{\mu\varepsilon}$

Lossless propagation constant: $k = \omega\sqrt{\mu\,\varepsilon} = \dfrac{2\pi}{\lambda}$

Wavelength: $\lambda = \dfrac{v}{f}$

Phase velocity: $v = \dfrac{1}{\sqrt{\mu\,\varepsilon}} = \dfrac{c}{\sqrt{\varepsilon_r}}$

Intrinsic Impedance: $\eta = \sqrt{\dfrac{\mu}{\varepsilon}}$

Poynting Vector: $\vec{S} = \vec{E} \times \vec{H}$

Time average power density: $S_{av} = \dfrac{1}{2}\mathrm{Re}\left\{\vec{E} \times \vec{H}^*\right\}$

Snell's Law: $n_i \sin(\theta_i) = n_t \sin(\theta_t)$

Index of refraction: $n = \sqrt{\varepsilon_r} = \sqrt{\varepsilon_r - j\dfrac{\sigma}{\omega\,\varepsilon_o}}$

Brewster's Angle:
$$\tan(\theta_B) = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$

Reflection Coefficients: $\Gamma_\perp = \dfrac{\vec{E}_r}{\vec{E}_i} = \dfrac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$

$$\Gamma_{//} = \frac{\vec{E}_r}{\vec{E}_i} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

Transmission coefficients $\tau_{\perp} = \frac{\vec{E}_t}{\vec{E}_i} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$

$$\tau_{//} = \frac{\vec{E}_t}{\vec{E}_i} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$$

$$\tau_{\perp} = 1 + \Gamma_{\perp}$$

$$\tau_{//} = \left(1 + \Gamma_{//}\right)\frac{\cos\theta_i}{\cos\theta_t}$$

Reflectivity (power reflection) $R_{\perp} = \frac{P_r}{P_i} = \left|\Gamma_{\perp}\right|^2$ $R_{//} = \frac{P_r}{P_i} = \left|\Gamma_{//}\right|^2$

Antenna Radiation Equation: $\vec{A}(R) = \frac{\mu}{4\pi}\int \vec{J}\,\frac{e^{-jk\left|\vec{R}-\vec{R}'\right|}}{\left|\vec{R}-\vec{R}'\right|}\,dv'$

Directivity:
$$D = \frac{\text{maximum power density}}{\text{power density of an isotropic radiator}} = \frac{S_{\max}}{S_{av}} = \frac{S_{\max}}{\left(\frac{P_{rad}}{4\pi r^2}\right)} = \frac{F_{\max}}{F_{av}} = \frac{1}{\frac{1}{4\pi r^2}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} F(\theta,\phi)}$$

$$D = \frac{F_{\max}}{F_{av}} = \frac{1}{\frac{1}{4\pi r^2}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} F(\theta,\phi)\, r^2 \sin^2(\theta)\, d\theta\, d\phi}$$

Gain:
$$G = \frac{S_{\max}}{\left(\frac{P_t}{4\pi r^2}\right)} = \xi D$$

Efficiency:
$$\xi = \frac{P_{rad}}{P_t}$$

Far-Field Hertzian Dipole equations:

$$\vec{E} = \hat{\theta}\; j\frac{I_o\, l\, k\, \eta_o}{4\pi}\left(\frac{e^{-jkR}}{R}\right)\sin(\theta)$$

$$\vec{H} = \hat{\phi}\; j\frac{I_o l\, k}{4\pi}\left(\frac{e^{-jkR}}{R}\right)\sin(\theta)$$

$$R_{rad} = 80\pi^2\left(\frac{l}{\lambda}\right)^2$$

Far-Field Half Wave Dipole Equations:

$$\vec{E} = \hat{\theta}\; j\,60\, I_o \left(\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right)\left(\frac{e^{-jkR}}{R}\right)$$

$$\vec{H} = \hat{\phi}\; j\frac{60\, I_o}{\eta_o}\left(\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right)\left(\frac{e^{-jkR}}{R}\right)$$

$$\vec{H} = \hat{\phi}\; j\frac{I_o l\, k}{4\pi}\left(\frac{e^{-jkR}}{R}\right)\sin(\theta)$$

$$R_{rad} = 73$$

Antenna Effective Area: $A_e = \dfrac{\lambda^2 D}{4\pi}$

Friis Transmission Formula: $\dfrac{P_{rec}}{P_t} = G_t\, G_r \left(\dfrac{\lambda}{4\pi R}\right)^2$

Antenna Array Pattern: $S = S_{\text{antenna}}\, F_a$

$$F_a = \left|\sum_{m=0}^{N-1} a_m\, e^{j\psi_m}\, e^{j\,m\,k\,d\cos\theta}\right|$$

Array Pattern for a uniform amplitude array antenna:

$$a_m = a\, e^{m\psi}$$

$$F_a = \frac{\sin^2\left[N\left(\frac{\psi}{2} + \frac{kd}{2}\cos\theta\right)\right]}{\sin^2\left[\left(\frac{\psi}{2} + \frac{kd}{2}\cos\theta\right)\right]}$$