8.7 Antenna Arrays

There are a variety of different benefits of antenna arrays. Here are a few of the most common advantages:

- Increasing the directivity of a simple antenna
- Concentrating the radiated power where you want it
- Eliminating the radiated power (or received power) where you don’t want it
- Electronically steering the main beam of a highly directional antenna

For the Hertzian dipole antenna,

\[ E_{ff} = \frac{\dot{\theta} \eta_0}{4\pi} I_0 \ell j k \frac{e^{-jkR}}{R} \sin \theta = \dot{\theta} E_0 \frac{e^{-jkR}}{R} \sin \theta \]

where we have lumped all the constants into \( E_0 \). Suppose we now have several such antennas, each having a different excitation current (magnitude and phase) arranged in a line. We call this an antenna array. We will generalize our far-field electric field for a single element so that our analysis can apply to any element type (not just the Hertzian dipole). The \( i \)th element will have electric field

\[ \overline{E_i}(R_i, \theta_i, \phi_i) = A_i \frac{e^{-jkR_i}}{R_i} \overline{f_e}(\theta_i, \phi_i) \]  

(8.38)

The terms in this expression are

- \( A_i = a_i e^{j\psi_i} = \) current weight or driving amplitude of the \( i \)th array element
- \( e^{-jkR_i}/R_i = \) spherical wave factor that we have for any antenna
- \( \overline{f_e}(\theta_i, \phi_i) = \) vector function that depends only on the observation angles \( (\theta_i, \phi_i) \) and is unique to a particular element type

The distance \( R_i \) is of course the distance from the \( i \)th element to the observation point. The angles have a similar definition, as shown.
The total field for this array is a sum over each element:

\[
\mathbf{E} = \sum_{i=0}^{N-1} A_i \frac{e^{-jkR_i}}{R_i} \mathbf{f}_e(\theta_i, \phi_i)
\]  \hspace{1cm} (8.39)

We now go back to our far-field assumption. From the figure, we can see that the far-field approximation suggests \( \theta_i = \theta \) and \( \phi_i = \phi \) for all \( i \). Also, \( R_1 = R - d \cos \theta \) for phase and \( R_1 = R \) for magnitude. For the \( i \)th element,

\[ R_i \approx R - id \cos \theta \]  

\[ R_i \approx R \]  

Our total field then becomes

\[
\mathbf{E} = \mathbf{f}_e(\theta, \phi) \frac{e^{-jkR}}{R} - \sum_{i=0}^{N-1} A_i e^{jkid \cos \theta} \]  \hspace{1cm} (8.40)

So, the total electric field can be written as the product of the single element radiation and an additional factor that takes into account the array. The power density for the array is is

\[
S_R(R, \theta, \phi) = S_{R,ff}(R, \theta, \phi)F_a(\theta)
\]  \hspace{1cm} (8.41)

where

\[
F_a(\theta) = \left| \sum_{i=0}^{N-1} A_i e^{jkid \cos \theta} \right|^2
\]  \hspace{1cm} (8.42)

We call \( F_a(\theta) \) the array factor for the pattern. It gives the shape of the radiation pattern due to the combination of the multiple elements independent of the shape of the individual element patterns. Often, the array factor dominates the behavior of the total radiation pattern. This is a very important result in array antenna theory: we can get the antenna pattern of the array by finding the array factor and then simply multiplying by the pattern of one individual element:

\[ [\text{Array Pattern}] = [\text{Element Pattern}] \times [\text{Array Factor}] \]  \hspace{1cm} (8.43)

### 8.7.1 Two Element Arrays

The two basic antenna design

**Analysis:** The individual antenna elements are known and we calculate the pattern that is produced.

**Synthesis:** The antenna pattern is known and we try and determine the individual antenna element locations and amplitudes that will produce this pattern.

Let’s look at two vertical Hertzian dipoles that are:

- separated by \( d = \frac{\lambda}{2} \)
• have equal amplitudes $a_0 = a_1$
• are out of phase by $\pi/2$ ($\phi_0 = 1, \phi_1 = \pi/2$)

The array factor becomes

$$F_a = \left| 1 + e^{-j\pi/2} e^{jkd \cos \theta} \right|^2$$

To simplify this we use

$$|1 + e^{jx}| = \left| 2e^{j\pi/4} \left( e^{-j\pi/4} + e^{j\pi/4} \right) \right|^2 = 4 \cos^2 \left( \frac{x}{2} \right)$$

The array factor becomes

$$F_a = 4 \cos^2 \left( \frac{\pi}{2} \cos \theta - \frac{\pi}{4} \right)$$

The total array power density is then given by

$$S_R(\theta, \phi) = 4S_{R,HD}(\theta, \phi) \cos^2 \left( \frac{\pi}{2} \cos \theta - \frac{\pi}{4} \right)$$

If we confine our attention to the $x - z$ plane, the pattern of the individual dipoles is isotropic, so the array pattern reduces to just the array factor. The array pattern has a maximum when

$$\frac{\pi}{2} \cos \theta - \frac{\pi}{4} = 0 \Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = \pm 60^\circ$$

and minimums when

$$\frac{\pi}{2} \cos \theta - \frac{\pi}{4} = -\frac{\pi}{2} \Rightarrow \theta = \pm 120^\circ$$

Figure 8.2: Polar plot of radiation pattern for a two element array example.
Next let’s look at an example of pattern synthesis. In this example we want to use two antennas to produce no radiation in the north/south directions and maximum radiation in the east/west direction.

Again we start with the array factor as given by

\[ F_a(\theta) = \left| 1 + a_1 e^{j\psi_1} e^{j\frac{2\pi d}{\lambda} \cos \theta} \right|^2 \]

We want \( F_a = 0 \) when \( \theta = \pm 90^\circ \) (north/south direction). Since \( \cos 90^\circ = 0 \) the array factor becomes

\[ F_a(\theta = 90^\circ) = \left| 1 + a_1 e^{j\psi_1} \right|^2 = 0. \]

This requires \( a_1 = a_o = 1 \) and \( e^{j\psi_1} = -1 \) or \( \psi_1 = \pi \). In order to have a maximum at \( \theta = 0 \) (\( \cos(0) = 1 \)) the array factor becomes

\[ F_a(\theta = 90^\circ) = \left| 1 + a_1 e^{j\psi_1} e^{j2\pi d\lambda} \right|^2 = 1 \]

resulting in

\[ e^{j\pi} e^{j2\pi d\lambda} = 1 \implies d = \frac{\lambda}{2} \]

The resulting array factor is

\[
F_a = \left| 1 - e^{j\pi \cos \theta} \right|^2 \\
= \left| \left( 2j e^{j\frac{\pi}{2}} \right) \left( e^{-j\frac{\pi}{2} \cos \theta} - e^{j\frac{\pi}{2} \cos \theta} \right) \right|^2 = 4 \sin^2 \left( \frac{\pi}{2} \cos \theta \right)
\]

![Array pattern synthesis example.](image)

If the phase were set at \( \psi_1 = 0 \) then the array pattern becomes \( F_a = 4 \cos^2 \left( \frac{\pi}{2} \cos \theta \right) \), and the picture rotates by \( 90^\circ \).