

2.10 Electric Properties of Materials

We classify materials based upon their *constitutive parameters*:

ϵ	electrical permittivity	F/m
μ	magnetic permeability	H/m
σ	conductivity	S/m

We will consider μ later in our discussion of magnetostatics. For now, let's focus on the other two.

Conductors and dielectrics are classified by how well they conduct current. Dielectrics are essentially insulators, meaning that the electrical resistance is high and can often be neglected. Good conductors on the other hand have low resistance. An ideal dielectric is a perfect insulator, whereas an ideal or perfect electric conductor (PEC) has zero resistance. Semiconductors are somewhere in between, and additionally have the property that they can be doped in such a way that current can be controlled and restricted to a given region.

2.10.1 Conductors

Conductivity relates current density to the electric field which supplies the force to move the charges. This relationship is

$$\vec{J} = \sigma \vec{E} \quad (2.102)$$

which looks a lot like Ohm's law (in fact, Ohm's law is a simplification of this). Given this definition, we see that a perfect dielectric has $\sigma = 0$ so that $\vec{J} = 0$. A perfect conductor, on the other hand, has $\sigma \rightarrow \infty$, which implies that to have finite current density we must have $\vec{E} = 0$ since $\vec{E} = \vec{J}/\sigma$. A perfect conductor is always an equipotential, meaning that the electric potential is the same everywhere on the conductor. For metals $\sigma \sim 10^7$ S/m, which is very large, so it is common practice to set $\vec{E} = 0$ inside a good conductor.

It is common in electromagnetic theory to model some current sources as *impressed currents*, meaning that we consider the current to be produced by an external source, and we treat it mathematically as a given source term in Maxwell's equations. In other words, we approximate the current as being independent of the fields we are solving for. In general, we have both induced conduction current and impressed current, so that

$$\vec{J} = \vec{J}_{induced} + \vec{J}_{impressed} \quad (2.103)$$

$$= \sigma \vec{E} + \vec{J}_{impressed} \quad (2.104)$$

2.10.2 Resistance

The definition of resistance is

$$R = \frac{V}{I} = \frac{-\int_l \vec{E} \cdot d\vec{\ell}}{\int_S \vec{J} \cdot d\vec{s}} = \frac{-\int_l \vec{E} \cdot d\vec{\ell}}{\int_S \sigma \vec{E} \cdot d\vec{s}} \quad (2.105)$$

This can be applied to determine the resistance of a given conductor geometry.

Example 2.5. Linear resistor

1. Assume a particular voltage.
2. Calculate the resulting electric field.
3. Calculate the current from the electric field.
4. Divide voltage by current to get resistance.

For a wire of cross sectional area A and length l carrying a uniformly distributed current density, the voltage along the wire is

$$V = - \int_{x_1}^{x_2} \bar{E} \cdot d\bar{\ell} \quad (2.106)$$

$$= - \int_{x_1}^{x_2} \hat{x} E_x \cdot \hat{x} dx \quad (2.107)$$

$$= E_x l \quad (2.108)$$

The total current is

$$I = \int_A \bar{J} \cdot d\bar{s} = \int_A \sigma \bar{E} \cdot d\bar{s} = \sigma E_x A \quad (2.109)$$

Using the definition of resistance, we obtain

$$R = \frac{V}{I} = \frac{E_x l}{\sigma E_x A} = \frac{l}{\sigma A} \quad (2.110)$$

For an integrated circuit interconnection, a typical process technology has interconnect lines that are $0.4\mu m$ wide and $0.6\mu m$ high. With copper as the metalization, the resistance per unit length of line is

$$R = \frac{1}{(5.8 \times 10^7)(.24\mu m^2)} \simeq 72 \Omega/mm \quad (2.111)$$

Example 2.6. Conductance of a Coaxial Cable

What is the conduction between the conductors of a coaxial cable? This time we will assume a current flowing between the two conductors I and then calculate the voltage from this assumed current. The current density for a section of coaxial cable of length l is

$$\bar{J} = \hat{r} \frac{I}{A} = \hat{r} \frac{I}{2\pi r l} \quad (2.112)$$

Since $\bar{J} = \sigma \bar{E}$, the electric field intensity is

$$\bar{E} = \hat{r} \frac{I}{2\pi \sigma r l} \quad (2.113)$$

We now calculate the voltage from the electric field,

$$V_{ab} = - \int_a^b \bar{E} \cdot d\bar{\ell} = - \int_a^b \frac{I}{2\pi \sigma l} \frac{\hat{r} \cdot \hat{r} dr}{r} \quad (2.114)$$

$$= \frac{I}{2\pi \sigma l} \ln \left(\frac{b}{a} \right) \quad (2.115)$$

The conductance per unit length is

$$\frac{G}{l} = \frac{1}{Rl} = \frac{I}{V_{abl}} = \frac{2\pi \sigma}{\ln(b/a)} \quad (2.116)$$

2.10.3 Dielectrics

To develop a simple model for a dielectric material, we will first assume that the dielectric is perfect, so that $\sigma = 0$ and $\bar{J} = 0$. At the microscopic level, an atom or molecule in the dielectric material consists of a positively charged nucleus and negatively charged electron cloud. Unlike a good conductor, for which the electrons can move freely through the material, the electrons in a dielectric only shift their positions relative to the nuclei as if the center of gravity of the cloud were attached to a spring with a given force constant. In the resting state, the cloud center is coincident with the nucleus center, leading to a neutral charge configuration. If an external electric field \bar{E}_{ext} is applied to the material, the center of the electron cloud will be displaced from its equilibrium value. While we still have charge neutrality, we can consider that there will be an electric field emanating from the positively charged nucleus and ending at the negatively charged cloud center. This process of creating electric *dipoles* within the material by applying an electric field is called *polarizing* the material.

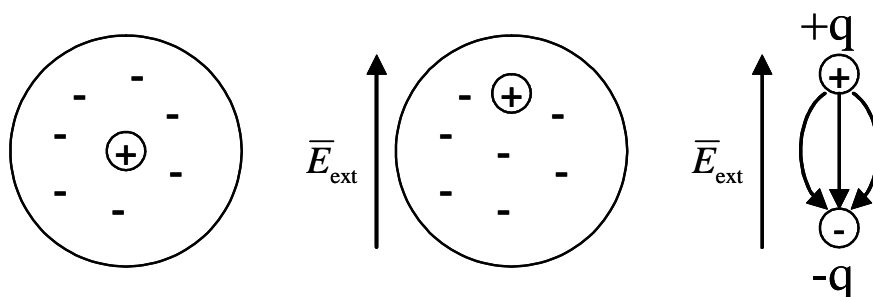


Figure 2.10: Electron cloud displaced by an externally applied electric field.

The induced electric field created by our new dipole is referred to as a *polarization* field, and it is weaker and in the opposite direction to \bar{E}_{ext} . If we think of sliding a dielectric slab in between the plates of a capacitor connected to a voltage source, additional charge must flow from the source onto the capacitor plates to maintain a constant voltage between the plates. So, the flux density increases to

$$\bar{D} = \epsilon_0 \bar{E} + \bar{P} \quad (2.117)$$

where $\epsilon_0 \simeq 8.854 \times 10^{-12}$ F/m is the permittivity of free space (vacuum) and \bar{P} is referred to as the *polarization vector* in the material. This quantity is proportional to the applied field strength, since the separation of the charges in the dielectric is more pronounced for stronger external fields. We can therefore write

$$\bar{P} = \epsilon_0 \chi_e \bar{E} \quad (2.118)$$

$$\bar{D} = \epsilon_0 \bar{E} + \epsilon_0 \chi_e \bar{E} = \epsilon_0 (1 + \chi_e) \bar{E} = \epsilon \bar{E} \quad (2.119)$$

$$\epsilon = \epsilon_0 \underbrace{(1 + \chi_e)}_{\epsilon_r} \quad (2.120)$$

where ϵ_r is the *relative permittivity* of the dielectric.

Some representative values for relative permittivity are

Material	Relative permittivity ϵ_r
Free space	1
Air	1.006
Polystyrene	2.6
Water	80
Barium titanate	1000 - 10,000