1.3.5 Standing Waves

When forward and reverse sinusoidal waves are both present on a transmission line, the two propagating waves add to form a standing wave pattern. If we apply a voltmeter to a transmission line instead of an oscilloscope, at high frequencies the voltmeter cannot respond rapidly enough to follow the \( \cos \omega t \) time variation, so what we actually measure is the standing wave pattern on the line. Standing waves are also helpful into gaining insight into transmission line phenomena for different types of loads.

Using the load reflection coefficient, the phasor voltage on a transmission line can be written as

\[
\tilde{V}(z) = V_o^+ e^{-j\beta z} + \Gamma_L V_o^+ e^{j\beta z}
\]  

(1.93)

In terms of the generalized reflection coefficient in Eq. (1.92), this becomes

\[
\tilde{V}(z) = V_o^+ e^{-j\beta z} [1 + \Gamma(z)]
\]  

(1.94)

To analyze the standing wave pattern, we look at the magnitude of the phasor:

\[
|\tilde{V}(z)| = |V_o^+| |1 + \Gamma(z)|
\]

(1.95)

\[
= |V_o^+| |1 + \Gamma_L e^{j\beta z}|
\]

\[
= |V_o^+| |1 + \Gamma_L e^{-j\beta z} e^{j2\beta z}|
\]

\[
= |V_o^+| \left[ (1 + |\Gamma_L| e^{j\beta z}) (1 + |\Gamma_L| e^{-j2\beta z}) \right]^{1/2}
\]

\[
= |V_o^+| \left[ 1 + 2|\Gamma_L| \cos (2\beta z + \theta_L) + |\Gamma_L|^2 \right]^{1/2}
\]  

(1.96)

We can understand this function graphically by going back to Eq. (1.95). If we plot the phasor voltage in the complex plane, we find that its value is equal to \( |V_o^+| \) on the real axis plus a complex number with magnitude \( |\Gamma_L||V_o^+| \) and phase \( 2\beta z \), as shown in Fig. 1.20. The magnitude of the phasor is equal to the distance from the origin to the sum of the two terms.

![Graphical representation of Eq. (1.95) and corresponding standing wave pattern.](image)

Figure 1.20: (a) Graphical representation of Eq. (1.95). (b) Corresponding standing wave pattern.

It is easy to see graphically that \( |\tilde{V}(z)| \) is bounded by

\[
|\tilde{V}(z)|_{\text{max}} = |V_o^+| (1 + |\Gamma_L|)
\]

(1.97)

\[
|\tilde{V}(z)|_{\text{min}} = |V_o^+| (1 - |\Gamma_L|)
\]

(1.98)

The phasor travels around the circle each time \( z \) changes by \( \pi/\beta \), which is equal to \( \lambda/2 \) or one half wavelength. Because the magnitude of the phasor voltage is the amplitude of the time-varying voltage in Eq. (1.58), \( |\tilde{V}(z)| \) is the envelope of the voltage along the transmission line as it oscillates in time.
**VSWR.** The voltage standing wave ratio (VSWR) is defined to be

\[
S = \frac{|\tilde{V}(z)|_{\text{max}}}{|\tilde{V}(z)|_{\text{min}}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}
\]

(1.99)

This quantity is useful because it is more easily measured on high frequency transmission lines than the time-varying voltage itself. For a matched load, \(\Gamma_L = 0\), so that the VSWR is equal to one and there is no standing wave on the transmission line.

### 1.3.6 Load Examples

**Matched load** (\(\Gamma_L = 0\)): Only a forward wave exists on the transmission line, and there is no standing wave.

![Standing wave pattern for a matched load.](image1.png)

**Open circuit** (\(\Gamma_L = 1\)): In this case, \(|\tilde{V}(z)|_{\text{min}}\) is zero and the VSWR is infinite. The standing wave pattern exhibits nulls spaced one half wavelength apart along the transmission line with a maximum at the load.

![Standing wave pattern for an open circuit load.](image2.png)

**Short circuit** (\(\Gamma_L = -1\)): \(|\tilde{V}(z)|_{\text{min}}\) is zero as with the open circuit load, and the VSWR is also infinite. In this case, however, there is a null instead of a maximum at the short circuit load.

![Standing wave pattern for a short circuit load.](image3.png)
1.3.7 Input Impedance

We need one more tool in order to analyze a complete sinusoidal steady state transmission system. Unlike the transient case, the impedance looking into the generator end depends on the entire transmission line and the load.

To understand impedance on a transmission line for a time-harmonic excitation, we can define a line impedance that is the ratio of the phasor voltage to the phasor current at a point on the line:

\[
Z_{\text{in}}(z) = \frac{\tilde{V}(z)}{\tilde{I}(z)} = \frac{V_o^+(1 + \Gamma_L e^{j2\beta z})}{V_o^+(1 - \Gamma_L e^{j2\beta z})/Z_o} = \frac{Z_o(1 + \Gamma(z))}{1 - \Gamma(z)} \quad (1.100)
\]

If the generator is located at \( z = -\ell \), then the input impedance seen by the source is \( Z_{\text{in}} = Z_{\text{in}}(-\ell) \). By substituting Eq. (1.91) for \( \Gamma \) and applying trigonometric identities to Eq. (1.101), the input impedance can be placed in an alternate form

\[
Z_{\text{in}} = Z_{\text{in}}(-\ell) = Z_o \frac{Z_L + jZ_o \tan \beta \ell}{Z_o + jZ_L \tan \beta \ell} \quad (1.103)
\]

This now allows us to analyze a complete time-harmonic transmission line system (Fig. 1.24).

![Transmission line system with sinusoidal excitation.](image)

In the steady state, as noted above the impedance seen by the source is \( Z_{\text{in}} \) as given by Eq. (1.103), so the voltage on the transmission line at the input port can be found using a voltage divider:

\[
\tilde{V}(-\ell) = \tilde{V}_g \frac{Z_{\text{in}}}{Z_g + Z_{\text{in}}} \quad (1.104)
\]

We can then find \( V_o^+ \) using Eq. (1.93), so that

\[
V_o^+ = \frac{\tilde{V}(-\ell)}{e^{j\beta \ell} + \Gamma_L e^{-j\beta \ell}} \quad (1.105)
\]

Once \( V_o^+ \) is known, all currents and voltages anywhere on the transmission line can be determined.